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Common Fixed Point Theorem with Two Self Maps in Complex Valued Metric Space

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Abstract—*Azam et. al introduced the notation of complex valued metric space and obtained common fixed point result for mappings in the context of complex valued metric space . In this paper, I prove the common fixed point theorem for a pair of mappings satisfying rational type contractive conditions in frame work of complex valued metric space. The prove results generalize and extended some of the known results in the literature.*

Keywords: *Contractive type mapping, Complex valued metric space, common fixed point, rational contraction.*
AMS Classification: 54H25, 47H10

INTRODUCTION -

The mathematical results regarding fixed point of contraction type mapping are useful for determining the existence and uniqueness of solutions to various mathematical problems. The concept of a complex valued metric space introduced by Azam, A and Fisher, B and Khan, M in 2011 [1], and obtained sufficient conditions for the existence of common fixed point of a pair of mappings satisfying a contractive type conditions . After the several authors studied many common fixed point results on complex valued metric space [2],[4],[6],[8],[9],[11],[12],[16].[17]. etc.

The aim of this paper is to establish a common fixed point theorems for two self maps satisfy a new contractive condition of rational type.

PRELIMINARIES

Definition 2.1 [1] let C be the set of complex number and let $Z_1 , Z_2 \in C$ as follows:

$$Z_1 \leq Z_2 \ _{iff} \ Re\ (Z_1) \leq Re(Z_2), Im\ (Z_1) \leq Im(Z_2) \ \dots .2.1.a$$

It follows that $Z_1 \leq Z_2$ if one of the following condition is satisfied

CN1. $Re\ (Z_1) = Re(Z_2)\ and\ Im\ (Z_1) = Im(Z_2)$

CN2 . $Re\ (Z_1) = Re(Z_2)\ and\ Im\ (Z_1) < Im(Z_2)$

CN3. $Re\ (Z_1) < Re(Z_2)\ and\ Im\ (Z_1) = Im(Z_2)$

CN4. $Re\ (Z_1) < Re(Z_2)\ and\ Im\ (Z_1) < Im(Z_2)$

In particular $Z_1 \lneqq Z_2$ if $Z_1 \neq Z_2$ and one of CN1, CN3, CN4 is satisfies and if $Z_1 < Z_2$

then only CN4 is satisfied that

Remark 2.2

1. $a, b \in R\ and\ a \leq b \Rightarrow aZ \leq bZ$

$$\forall\ Z \in C$$

2. $0 \leq Z_1 \nleq Z_2 \Rightarrow |Z_1| < |Z_2|$

3. $Z_1 \leq Z_2$ and $Z_2 < Z_3 \Rightarrow Z_1 < Z_3$

Definition 2.3 Let X be a non-empty set, & C be the set at complex numbers, suppose that the mapping d: $X \times X \rightarrow C$ satisfies the following conditions

(i) $0 \leq d(x,y)$,$\forall\, x,y\, \in X$ & $d\,(x,y) = 0\; iff\; x = y$

(ii) $d(x,y) = d(y,x), \forall\, x,y\, \in X$

(iii) $d(x,y) \leq d(x,z) + d(z,y), \forall\, x,y \in X$

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

Example 2.4

Let $X = C$, Define the mapping $d: X \times X \rightarrow C$ by,

$d(Z_1, Z_2) = 2i\,|Z_1 - Z_2|$ for all

$Z_{1,}\, Z_2 \in X$. Then (X,d) is a complex valued metric space.

Definition 2.5 Let (X,d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converge to x iff

$|\, d(x_n, x)| \rightarrow 0$ as $n \rightarrow \infty$

Where $d(x_n, x) < c$ for all $n \geq N$

Definition 2.6 Let (X,d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence .

$|\, d(x_n, x_{n+m})| \rightarrow 0$ as $n \rightarrow \infty$ where $m \in N$. where $d(x_n, x_{n+m}) < c$

3. Main Result : Let (X,d) be a complete complex valued metric space and let the mapping $S,T : X \rightarrow X$ satisfy the condition.

$$d(Sx,Ty) \leq \alpha\, \frac{d(x,Ty)d(y,Ty)+[d(x,y)]^2}{d(x,y)} + \beta\, \frac{d(x,Sx)d(x,Ty)+[d(x,y)]^2}{d\,(x,y)} + \gamma\, \frac{d(x,Sx)d(y,Ty)}{d(x,y)} + \delta\,[\,d(\,x,Sx) + d(y,Ty)] + \eta[d(x,Sx) + d(x,Ty)] + \mu d\,(x,y).$$

for all $x,y \in X$ such that $x \neq y, d(x,y) \neq 0$

Where $\alpha,\beta,\gamma,\delta,\eta,\mu$ are non negative reals with $\alpha + \beta + 2\gamma + 2\delta + \eta + \mu < 1$ or

$d(Sx,Ty) \leq 0$ if $d(x,y) = 0$ then S & T have a unique common fixed points.

Proof: Let x_0 be on a arbitrary points in X and define $x_{2k+2} = Sx_{2k+1}$

and $x_{2k+1} = Tx_k$ where $k = 0,1,2\ \dots\dots$

Then

$d(x_{2k+2}, x_{2k+1}) = d(Sx_{2k+1}, Tx_{2k})$,

$$\leq \alpha\, \frac{d(x_{2k+1}, Tx_{2k})d(x_{2k,}Tx_{2k}) + [d(x_{2k+1}, x_{2k})]^2}{d(x_{2k+1}, x_{2k})} + \beta\, \frac{d(x_{2k+1}, Sx_{2k+1})d(x_{2k+1}, Tx_{2k}) + [d(x_{2k+1}, x_{2k})]^2}{d(x_{2k+1}, x_{2k})}$$

$$+ \gamma\, \frac{d(x_{2k+1}, Sx_{2k+1})d(x_{2k}, Tx_{2k})}{d\,(x_{2k+1}, x_{2k})} + \delta[d(x_{2k+1}, Sx_{2k+1}) + d(x_{2k}, Tx_{2k})]$$

$+\eta\left[d(x_{2k}, Sx_{2k+1}) + d\left(x_{2k+1,}Tx_{2k}\right)\right] + \mu d(x_{2k+1}, x_{2k})$

$$\leq \alpha \frac{d(x_{2k+1}, x_{2k+1})d\left(x_{2k,}x_{2k+1}\right) + [d(x_{2k+1}, x_{2k})]^2}{d(x_{2k+1}, x_{2k})} + \beta \frac{d(x_{2k+1}, x_{2k+2})d(x_{2k+1}, x_{2k+1}) + [d(x_{2k+1}, x_{2k})]^2}{d(x_{2k+1}, x_{2k})}$$

$$+ \gamma \frac{d(x_{2k+1}, x_{2k+2})d(x_{2k}, x_{2k+1})}{d\ (x_{2k+1}, x_{2k})} + \delta[d(x_{2k+1}, x_{2k+2}) + d(x_{2k}, x_{2k+1})]$$

$$+ \eta\left[d(x_{2k}, x_{2k+2}) + d\left(x_{2k+1,}x_{2k+1}\right)\right] + \mu d(x_{2k+1}, x_{2k})$$

$$\leq \alpha d(x_{2k+1}, x_{2k}) + \beta d(x_{2k+1}, x_{2k}) + \gamma d(x_{2k+1}, x_{2k+2}) + \delta[d(x_{2k+1}, x_{2k+2}) + d(x_{2k}, x_{2k+1})]$$

$+\eta d\ (x_{2k}, x_{2k+2}) + \mu d(x_{2k+1}, x_{2k})$

$d(x_{2k+2}, x_{2k+1}) \leq$

$\left(\alpha + \beta + \delta + \eta + \mu\right) d(x_{2k+1}, x_{2k}) + \left(\gamma + \delta + \eta\right) d\ (x_{2k+1}, x_{2k+2})$

$d(x_{2k+2}, x_{2k+1}) \leq \frac{\alpha+\beta+\delta+\eta+\mu}{1-\gamma-\delta-\eta} d(x_{2k+1}, x_{2k})$

because $\alpha + \beta + 2\gamma + 2\delta + \eta + \mu < 1$

So that

$\left| d(x_{2k+2}, x_{2k+1}) \right| \leq Q\, d \left| (x_{2k+1,}x_{2k}) \right|$

As by triangle inequality

$$\left| d(x_{2k+1}, x_{2k+2}) \right| \leq \mathrm{d}[\left(x_{2k+1,}x_{2k}\right) \right| + \left| \mathrm{d}(x_{2k}, x_{2k+2}) \right|$$

Similary

$d(x_{2k+3}, x_{2k+2}) = d(Sx_{2k+2}, Tx_{2k+1})$

$$\leq \alpha \frac{d(x_{2k+2}, Tx_{2k+1})d\left(x_{2k+1,}Tx_{2k+1}\right) + [d(x_{2k+2}, x_{2k+1})]^2}{d(x_{2k+2}, x_{2k+1})} + \beta \frac{d(x_{2k+2}, Sx_{2k+2})d(x_{2k+2}, Tx_{2k+1}) + [d(x_{2k+2}, x_{2k+1})]^2}{d(x_{2k+2}, x_{2k+1})}$$

$$+ \gamma \frac{d(x_{2k+2}, Sx_{2k+2})d(x_{2k+1}, Tx_{2k+1})}{d\ (x_{2k+2}, x_{2k+1})} + \delta[d(x_{2k+2}, Sx_{2k+2}) + d(x_{2k+1}, Tx_{2k+1})]$$

$$+ \eta\left[d(x_{2k+1}, Sx_{2k+2}) + d\left(x_{2k+2,}Tx_{2k+1}\right)\right] + \mu d(x_{2k+2}, x_{2k+1})$$

$\leq \alpha \frac{d(x_{2k+2}, x_{2k+2})d\left(x_{2k+1,}x_{2k+2}\right) + [d(x_{2k+2,}x_{2k+1})]^2}{d(x_{2k+2,}x_{2k+1})} + \beta \frac{d(x_{2k+2,}x_{2k+3})d(x_{2k+2,}x_{2k+2}) + [d(x_{2k+2,}x_{2k+1})]^2}{d(x_{2k+2,}x_{2k+1})} + \gamma \frac{d(x_{2k+2,}x_{2k+3})d(x_{2k+1,}x_{2k+2})}{d\ (x_{2k+2,}x_{2k+1})} +$
$\delta[d(x_{2k+2}, x_{2k+3}) + d(x_{2k+1}, x_{2k+2})] + \eta\left[d(x_{2k+1}, x_{2k+3}) + d\left(x_{2k+2,}x_{2k+2}\right)\right] + \mu d(x_{2k+2}, x_{2k+1})$

$$d(x_{2k+3}, x_{2k+2})$$

$\leq \alpha d(x_{2k+2}, x_{2k+1}) + \beta d(x_{2k+2}, x_{2k+1}) + \gamma d(x_{2k+2}, x_{2k+3}) + \delta[d(x_{2k+2}, x_{2k+3}) + d(x_{2k+1}, x_{2k+2})] + \eta d\ (x_{2k+1}, x_{2k+3}) +$
$\mu d(x_{2k+2}, x_{2k+1})$

$$\leq \left(\alpha + \beta + \delta + \eta + \mu\right) d(x_{2k+2}, x_{2k+1}) + \left(\gamma + \delta + \eta\right) d\ (x_{2k+2}, x_{2k+3})$$

$d(x_{2k+3}, x_{2k+2})$

$\leq \frac{\alpha+\beta+\delta+\eta+\mu}{1-(\gamma+\delta+\eta)} d(x_{2k+2}, x_{2k+1})$

$$d(x_{2k+3}, x_{2k+2}) \leq Q\ d(x_{2k+2}, x_{2k+2})$$

Where

$\left| d(x_{2k+3}, x_{2k+2}) \right| \le Q \left| [d((x_{2k+1}, x_{2k+2})] \right|$
by triangle inequality.

$\left| d(x_{2k+2}, x_{2k+3}) \right| \le d \left| (x_{2k+2}, x_{2k+1}) \right| + \left| d(x_{2k+1}, x_{2k+3}) \right|$

$\left| d(x_{n+1}, x_{n+2}) \right| \le Q \left| d(x_n, x_{n+1}) \right|$

$\le \ldots\ldots \le Q^{n+1} \left| d(x_0, x_1) \right|$

So that $m > n$.

As by triangle inequality.

$$\left| d(x_n, x_m) \right| \le \left| d(x_n, x_{n+1}) \right| + \left| d(x_{n+1}, x_{n+2}) \right| + \left| d(x_{n+2}, x_{n+3}) \right| + \cdots . + \cdots . + \left| d(x_{m-1}, x_m) \right|$$

$\le (Q^n + Q^{n+1} + \cdots . + Q^{n-1}) \left| d(x_0, x_1) \right|$

$\le \frac{Q^n}{1-Q} \left| d(x_0, x_1) \right|$

Hence $\left| d(x_m, x_n) \right| \le \frac{Q^n}{1-Q} \left| d(x_0, Tx_0) \right|$

as $m, n \to \infty$

This implies that $\{x_n\}$ is a Couchy sequence in X since X is complete $\exists$ some.

$p \in X$ such that $S_n \to p$ as $n \to \infty$

Suppose on the contrary that $p \neq Tp$

So that $d(p, Tp) = Z > 0$.

$d(p, Tp) = Z \le d(p, x_{2k+2}) + (x_{2k+2}, Tp)$

$\le d(p, x_{2k+2}) + d(Sx_{2k+1}, Tp)$

$\le d(p, x_{2k+2}) + \alpha \frac{d(x_{2k+1}, Tp)d(p,Tp) + [d(, x_{2k+1}, p)]^2}{d(x_{2k+1}, p)} + \beta \frac{d(x_{2k+1}, Sx_{2k+1})d(x_{2k+1}, Tp) + [d(x_{2k+1}, p)]^2}{d(x_{2k+1}, p)} + \gamma \frac{d(x_{2k+1}, Sx_{2k+1})d(p, Tp)}{d(x_{2k+1}, p)} +$
$\delta[d(x_{2k+1}, Sx_{2k+1}) + d[p, Tp)] + \eta[d(p, Sx_{2k+1}) + d(Sx_{2k+1}, Tp)] + \mu d(x_{2k+1}, p)$

$$\le d(p, x_{2k+2}) + \alpha \frac{d(x_{2k+1}, Tp)Z + [d(, x_{2k+1}, p)]^2}{d(x_{2k+1}, p)} + \beta \frac{d(x_{2k+1}, x_{2k+2})d(x_{2k+1}, Tp) + [d(x_{2k+1}, p)]^2}{d(x_{2k+1}, p)} + \gamma \frac{d(x_{2k+1}, x_{2k+2})Z}{d(x_{2k+1}, p)} + \delta[d(x_{2k+1}, x_{2k+2}) + Z] + \eta[d(p, x_{2k+2}) + d(x_{2k+2}, Tp)] + \mu d(x_{2k+1}, p)$$

So that

$\left| d(p, Tp) \right| = \left| Z \right| \le$

$$\left| d(p, x_{2k+2}) \right| + \alpha \frac{\left| d^2(v, x_{2k+2}) \right|}{\left| d(x_{2k+1}, x_{2k+2}) \right| + \left| d(x_{2k+1}, v) \right|} + \beta \frac{\left| d^2(x_{2k+1}, Tv) \right|}{\left| d(x_{2k+1}, Tv) \right| + \left| d(x_{2k+1}, v) \right|} +$$

$$\gamma \frac{\left| Z^2 \right|}{\left| Z \right| + \left| d(x_{2k+1}, v) \right|} + \delta \left[\left| d(x_{2k+1}, x_{2k+2}) \right| + \left| Z \right| \right] + \square \left[\left| d\,(x_{2k+1}, Tv) \right| + \left| d(v, x_{2k+2}) \right| \right] +$$

$\mu \left| d(x_{2k+1}, v) \right|$

Which or mapping $n \to \infty$ therefore

$$\left| d(p, Tp) \right| = 0$$

Which is contradiction so that

$p = Tp$

Similarly we show that

$p = Sp$ thus implies that p is fixed point.

Uniqueness

Let q in X be another common fixed point of S & T, then

$d(q,p) = d(Sq, Tp)$

$$\leq \alpha \frac{d(q,Tp)d(p,Tp)+[d(q,p)]^2}{d(q,p)} + \beta \frac{d(q,Sq)d(q,Tp)+[d(q,p)]^2}{d(q,p)} + \gamma \frac{d(q,Sq)d(p,Tp)}{d(q,p)} + \delta[d(q,Sq) + d\,[p,Tp)] + \eta[d\,(p,Sq) + d(q,Tp)] +$$

$\mu d\,(q,p)$

So $d(q,p) \leq \alpha d(q,p) + \beta d\,(q,p) + \eta[d\,(p,q) + d(q,p)] + \mu d\,(q,p)$

$$d(q,p) \leq \left(\alpha + \beta + 2\eta + \mu\right) d(p,q)$$

So $d(q,p) \leq \rho\, d(p,q)$

$$\therefore \rho = \left(\alpha + \beta + 2\eta + \mu\right) < 1$$

So $p = q$

Which proves the uniqueness of common fixed point.

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